

Regularization of superstring amplitudes and a cancellation of divergences in superstring theory

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Abstract

For a calculation of divergent fermion string amplitudes a regularization procedure invariant under the supermodular group is constructed. By this procedure superstring amplitudes of an arbitrary genus are calculated using both partition functions and superfield vacuum correlators computed early. A finiteness of superstring amplitudes and related topics are discussed.

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1 Introduction

For a long time the central matter of multi-loop calculations in superstring theory [1] was focused on partition functions and of field vacuum correlators [2—9]. Essentially, difficulties were found for Ramond strings where the above values can not be derived by an obvious extension of boson string results [10]. A method calculating the discussed values has been developed in [6, 8]. By this method the partition functions and the superfield vacuum correlators were calculated explicitly [8, 9] in terms of super-Schottky group [7, 8] parameters for all both the Ramond strings and the Neveu-Schwarz ones. In a calculation of superstring amplitudes through the above values the main difficulty is due to every fermion string amplitude is divergent. Removing the above difficulty is considered in this paper.

As it is usually, a genus- n closed superstring amplitude $A_{n,m}$ with m legs is given by

$$A_{n,m} = \int \prod_N dq_N d\bar{q}_N \prod_{r=1}^m dt_r d\bar{t}_r \sum_{L,L'} Z_{L,L'}^{(n)}(\{q_N, \bar{q}_N\}) E_{L,L'}(\{t_r, \bar{t}_r\}) \quad (1)$$

where the overline denotes the complex conjugation and L (L') labels superspin structures of right (left) superfields defined on the complex $(1|1)$ supermanifolds [11]. Every superspin structure $L = (l_1, l_2)$ presents a superconformal extension of the $(l_1, l_2) = \bigcup_{s=1}^n (l_{1s}, l_{2s})$ ordinary spin one [12]. The genus- n theta function characteristics (l_1, l_2) can be restricted by $l_{is} \in (0, 1/2)$. As the q_N moduli, $(3n-3|2n-2)$ of $(3n|2n)$ super-Schottky modular parameters are used, the rest being in a number of $(3|2)$, is fixed commonly to all the genus- n supermanifolds by a super-Möbius transformation. Partition functions $Z_{L,L'}^{(n)}(\{q_N, \bar{q}_N\})$ are calculated from equations [6, 8] expressing that the superstring amplitudes are independent of a choice of two-dim. metrics and of a gravitino field. The vacuum expectation $E_{L,L'}(\{t_r, \bar{t}_r\})$ of the vertex product depends on the $\{t_r\}$ set of vertex supercoordinates and on $\{q_N, \bar{q}_N\}$, as well. We map the supermanifolds by the supercoordinate $t = (z|\theta)$ where z is a local complex coordinate and θ is its odd partner. Every t_r integration in (1) is performed over the supermanifold. Each of the (L, L') terms is covariant [13] under the supermodular group, moduli being integrated over the fundamental domain calculated in [13]. Among other things, the above fundamental domain depends on L through terms proportional to odd super-Schottky parameters because, generally, the supermodular changes of moduli and of supercoordinates depend on superspin structure [13]. For the same reason, the sum over (L, L') in (1) calculated with $\{q_N\}$ common to all the superspin structures is non-covariant under the group considered.

The integration of every superspin structure in (1) is divergent [14] due to a degeneration of Riemann surfaces. It is expected usually [2, 3, 15] that all the divergences are canceled in the whole amplitude $A_{n,m}$, but, in any case, a correct calculation of $A_{n,m}$ requires a regularization procedure. The regularization procedure is bound to ensure the invariance of the superstring amplitudes under supermodular group. At the same time, a cutoff of modular integrals proposed in [16] violates the above group. Specifically, in [16] the non-split property of the supermanifolds [13] is ignored. There is no a test for the supermodular invariance to be restored after removing the cutoff [16] once the integrals in (1) are calculated and the summation over superspin structures is performed. In contrast, we build a manifestly supermodular regularization of (1) multiplying every term of the sum by a supermodular invariant function. Besides the modular integrals, we regularize integrals over z_j that are ill defined when all the vertices tend to coincide, or, alternatively, all they are moved away from each other. As a

by-product, we construct a supermodular covariant sum over superspin structures. A popular opinion is that all the non-integrable singularities are cancelled locally in sum of this kind, but it has not really a firm basis. Indeed, the above local cancellation of the divergences requires [2, 3] vanishing the supercovariant sum over superspin structures of the partition functions. It might be reasonable [17], if this sum is essentially a product of a function holomorphic in moduli by an anti-holomorphic one. But except the genus-one case, such is not the case because the supermanifold period matrices of a genus- $n > 1$ depend [5, 8] on L . Further still, in this condition a local nullification of the discussed sum of the partition functions is not necessary for a vanishing [3] of the whole vacuum amplitude. Moreover, even though [15], it is not established that divergences are locally cancelled [15] in the multi-loop Green-Schwarz amplitudes because in [15] the non-split property of the supermanifolds [13] is not taken into account. In our approach a divergence cancellation in (1) are studied once the integrals are calculated.

Poles and of threshold singularities of $A_{n,m}$ in a given reaction channel j are due to an integration over a suitable nodal domain. As it is usual [18], each of the above integrals is calculated at $Re E_j^2 < 0$ where E_j is a center mass energy in the channel considered. Being divergent itself at $Re E_j^2 > 0$, it is extended over $Re E_j^2 > 0$ by an analytical continuation in E_j^2 . Since there is no a region in the $\{E_j^2\}$ space where all the discussed integrals would be finite together and since a boundaries of the nodal domains are changed under the supermodular group, the supermodular invariance of the considered procedure may seem doubtful. We improve this procedure and clarify supermodular invariance of it.

We expect that amplitudes of an emission of a longitudinal polarized gauge boson vanish in our scheme as it is required by the gauge symmetry. In addition, the 0-, 1-, 2- and 3- point functions of massless superstring modes are nullified in line with [3]. A full study of the above matters is planned in the next future.

The regularization procedure for the modular integrals is considered in Sec.2. It is partly overlapped with [19] where a regularization of those was proposed. In Sec.3 regularized expressions for superstring amplitudes are given. In Sec.4 the cancellation of divergences in superstring amplitudes, the gauge symmetry and non-renormalization theorems [3] are discussed.

2 Regularization of modular integrals

A construction of desired supermodular invariant functions of moduli is complicated due to supermodular changes of q_N depend on the superspin structure [13]. To build the desired function we perform a singular $t \rightarrow \hat{t} = (\hat{z}|\hat{\theta})$ superholomorphic transformation [19] to a new parameterization P_{split} where transition groups contain no Grassmann parameters:

$$z = f_L(\hat{z}) + f'_L(\hat{z})\hat{\theta}\xi_L(\hat{z}), \quad \theta = \sqrt{f'_L(\hat{z})} \left[\left(1 + \frac{1}{2}\xi_L(\hat{z})\xi'_L(\hat{z}) \right) \hat{\theta} + \xi_L(\hat{z}) \right], \quad f_L(\hat{z}) = \hat{z} + y_L(\hat{z}). \quad (2)$$

Here the "prime" symbolizes \hat{z} -derivative, $\xi_L(\hat{z})$ is a Grassmann function and the $y_L(\hat{z})$ function is proportional to odd modular parameters. Rounds about (A_s, B_s) -cycles are given by super-Schottky transformations $(\Gamma_{a,s}(l_{1s}), \Gamma_{b,s}(l_{2s}))$, every A_s -cycle being associated with a suitable Schottky circle. In this case one see [7, 8] that $\Gamma_{a,s}(l_{1s} = 0) = I$, $\Gamma_{a,s}^2(l_{1s} = 1/2) = I$, but $\Gamma_{a,s}(l_{1s} = 1/2) \neq I$. So a square root cut on the z -plane appears for every $l_{1s} \neq 0$ with endcut points to be inside corresponding Schottky circles. In the P_{split} parameterization the same

rounds are associated with transformations $(\hat{\Gamma}_{a,s}(l_{1s}), \hat{\Gamma}_{b,s}(l_{2s}))$. In this case

$$\Gamma_{b,s}(l_{2s})(t) = t \left(\hat{\Gamma}_{b,s}(l_{2s})(\hat{t}) \right), \quad \Gamma_{a,s}(l_{1s})(t) = t^{(s)} \left(\hat{\Gamma}_{a,s}(l_{1s})(\hat{t}) \right) \quad (3)$$

where $t^{(s)}(\hat{t})$ is obtained by 2π -twist of $t(\hat{t})$ on the complex \hat{z} -plane about the Schottky circle assigned to a particular handle s . Both $\hat{\Gamma}_{a,s}(l_{1s})$ and $\hat{\Gamma}_{b,s}(l_{2s})$ do not contain Grassmann modular parameters. Since the super-Schottky transformations depend, among other things, on $(2n-2)$ Grassmann moduli, the transition functions in (2) depends on $(2n-2)$ Grassmann parameters $(\lambda_j^{(1)}, \lambda_j^{(2)})$ where $j = 1 \dots n-1$. Eqs.(3) are given explicitly in [19]. Kindred equations were used in [13] to calculate the acting of supermodular transformations on supercoordinates and on modular parameters. Unlike those in [13], eqs.(3) can be satisfied only if the transition functions in (2) have poles in a fundamental region of \hat{z} -plane. We take ¹ them possessing $(n-1)$ poles \hat{z}_j of an order 2. For even superspin structures we choose the above poles among n zeros of the fermion Green function $R_L^f(\hat{z}, \hat{z}_0)$ calculated for zero odd moduli.² For odd superspin structures the poles can be chosen by a similar way [19]. We take \hat{z}_0 common to all superspin structures. In this case supermodular changes of $(\lambda_j^{(1)}, \lambda_j^{(2)})$ are independent of the superspin structure and the supermodular group in the P_{split} representation is mainly reduced to the ordinary modular one. The singular parts of (2) are determined by a condition that above modular group is isomorphic to the supermodular one in the super-Schottky parameterization. In this case one obtains [19] near every pole $\hat{z}_j(\hat{z}_0; L)$ that

$$\begin{aligned} \xi_L(\hat{z}) &\approx \frac{[1 + \xi_L(\hat{z})\xi_L'(\hat{z})]}{R_L^f(\hat{z}, \hat{z}_0)} \left[\frac{\lambda_j^{(2)}}{[R_L^f(\hat{z}, \hat{z}_0)]} \frac{\partial R_L^f(\hat{z}, \hat{z}_0)}{\partial \hat{z}_0} + \lambda_j^{(1)} \right] + \frac{\lambda_j^{(1)}\lambda_j^{(2)}\xi_L(\hat{z})}{2[R_L^f(\hat{z}, \hat{z}_0)]^2} \frac{\partial^2 \ln[R_L^f(\hat{z}, \hat{z}_0)]}{\partial \hat{z} \partial \hat{z}_0}, \\ f_L(\hat{z}) &\approx \frac{\lambda_j^{(2)}\xi_L(\hat{z})f_L'(\hat{z})}{[R_L^f(\hat{z}, \hat{z}_0)]^2} \frac{\partial R_L^f(\hat{z}, \hat{z}_0)}{\partial \hat{z}_0} + \frac{\lambda_j^{(1)}\xi_L(\hat{z})f_L'(\hat{z})}{R_L^f(\hat{z}, \hat{z}_0)} \end{aligned} \quad (4)$$

where the "prime" symbol denotes $\partial_{\hat{z}}$. The calculation of both $y_L(\hat{z})$ and $\xi_L(\hat{z})$ is quite similar to that in Sec. 3 of [13]. For this purpose we start with relations

$$\xi_L(\hat{z}) = - \int_{C(\hat{z})} G_{gh}^{(f)}(\hat{z}, \hat{z}'; L) \xi_L(\hat{z}') \frac{d\hat{z}'}{2\pi i}, \quad y_L(\hat{z}) = \int_{C(\hat{z})} G_{gh}^{(b)}(\hat{z}, \hat{z}') y_L(\hat{z}') \frac{d\hat{z}'}{2\pi i} \quad (5)$$

where an infinitesimal contour $C(z)$ gets around z -point in the positive direction. Both Green functions $G_{gh}^{(f)}(\hat{z}, \hat{z}'; L)$ and $G_{gh}^{(b)}(\hat{z}, \hat{z}')$ are defined³ in [13]. We deform the $C(\hat{z})$ to surround both the \hat{z}_j poles and the Schottky circles together with the cuts presenting for $l_{1s} \neq 0$. The integrals along Schottky circles and along the cuts are transformed by (3) to the form similar to [13]. The integrals around the poles are calculated using (4). Relations for modular parameters appear just as in Sec.4 of [13] because of $G_{gh}^{(f)}(\hat{z}, \hat{z}'; L)$ and $G_{gh}^{(b)}(\hat{z}, \hat{z}')$ receive additional terms under Schottky transformations on \hat{z} -plane.⁴ The resulted equations [19] determine both $y_L(\hat{z})$ and $\xi_L(\hat{z})$ together with complex super-Schottky moduli q_N up to SL_2 transformations of t .

¹An another choice of the poles is discussed in [19].

²See Sec. 4 of [8] where $R_L^f(\hat{z}, \hat{z}_0)$ is denoted as $R_f(z, z')$.

³ $G_{gh}^{(f)}(\hat{z}, \hat{z}'; L)$ is denoted in [13] as $G_{gh}^{(f)}(\hat{z}, \hat{z}')$, the explicit L dependence being omitted.

⁴See eqs.(23) in [13].

The partition functions $\hat{Z}_{L,L'}^{(n)}(\{\hat{q}_N, \overline{\hat{q}_N}\})$ in the P_{split} representation are given by

$$\hat{Z}_{L,L'}^{(n)}(\{\hat{q}_N, \overline{\hat{q}_N}\}) = F_L(\{\hat{q}_N\}) \overline{F_{L'}(\{\hat{q}_N\})} Z_{L,L'}^{(n)}(\{q_N, \overline{q_N}\}) \quad (6)$$

where $F_L(\{\hat{q}_N\})$ is Jacobian of the considered transformation and $q_N = q_N(\{\hat{q}_N\}; L)$. Further, $\{\hat{q}_N\} = \{\hat{q}_{ev}, \lambda_j^{(1)}, \lambda_j^{(2)}\}$, where \hat{q}_{ev} are even complex moduli in a number of $(3n - 3)$. The supermodular invariant sum over superspin structures is constructed by doing the $(\{\hat{q}_N\}, \hat{t})$ set to be common to all superspin structures. Superstrings are non-invariant under the considered singular transformations (2). That is why expressions for the amplitudes in the P_{split} parameterization (once integrations over $\{\lambda_j^{(1)}, \lambda_j^{(2)}\}$ are performed) differ from those in [2] where a split property of the supermanifolds is assumed (for more details, see [19]).

The desired supermodular invariant function $Y(\{\hat{q}_N, \overline{\hat{q}_N}\}; \hat{z}_0, \overline{\hat{z}_0})$ is constructed by (6) as

$$Y(\{\hat{q}_N, \overline{\hat{q}_N}\}; \hat{z}_0, \overline{\hat{z}_0}) = \frac{[Y_1(\{\hat{q}_N, \overline{\hat{q}_N}\}; \hat{z}_0, \overline{\hat{z}_0})]^{2^{n-1}(2^n+1)}}{Y_2(\{\hat{q}_N, \overline{\hat{q}_N}\}; \hat{z}_0, \overline{\hat{z}_0})} \quad (7)$$

with $Y_1(\{\hat{q}_N, \overline{\hat{q}_N}\}; \hat{z}_0, \overline{\hat{z}_0}) \equiv Y_1$ and $Y_2(\{\hat{q}_N, \overline{\hat{q}_N}\}; \hat{z}_0, \overline{\hat{z}_0}) \equiv Y_2$ defined to be

$$Y_1 = \sum_{L \in \{L_{ev}\}} \hat{Z}_{L,L}^{(n)}(\{\hat{q}_N, \overline{\hat{q}_N}\}) \quad \text{and} \quad Y_2(\{\hat{q}_N, \overline{\hat{q}_N}\}; \hat{z}_0, \overline{\hat{z}_0}) = \prod_{L \in \{L_{ev}\}} \hat{Z}_{L,L}^{(n)}(\{\hat{q}_N, \overline{\hat{q}_N}\}) \quad (8)$$

where $\{L_{ev}\}$ is the set of $2^{n-1}(2^n + 1)$ even spin structures and $q_N = q_N(\{\hat{q}_N\}; L)$. Since both $Y_1(\{\hat{q}_N, \overline{\hat{q}_N}\})$ and $Y_2(\{\hat{q}_N, \overline{\hat{q}_N}\})$ receive the same factor under modular transformation of \hat{q}_N -parameters, the right side of (7) is invariant under supermodular transformations. In addition, it tends to infinity, if Riemann surfaces are degenerated. Indeed, if a particular handle, say s , become degenerated, the corresponding Schottky multiplier k_s tends to zero. In this case both the nominator and the denominator in (7) tend to infinity [8], but terms associated with $l_{1s} = 0$ have an additional factor $|k_s|^{-1} \rightarrow \infty$ in a comparison with those associated with non-zero l_{1s} . So $Y(\{\hat{q}_N, \overline{\hat{q}_N}\}; \hat{z}_0, \overline{\hat{z}_0}) \rightarrow \infty$. If a even spin structure of a genus- $n > 1$ is degenerated into odd spin structures, the partition functions tend to zero [9] and they do not vanish, if it is degenerated into even spin ones. So again $Y(\{\hat{q}_N, \overline{\hat{q}_N}\}; \hat{z}_0, \overline{\hat{z}_0}) \rightarrow \infty$. Hence to regularize the desired integrals one can introduce in the integrand (1) a multiplier

$$B_{mod}^{(n)}(\{q_N, \overline{q_N}\}; \hat{z}_0, \overline{\hat{z}_0}; \delta_0) = \{\exp[-\delta_0 Y(\{\hat{q}_N, \overline{\hat{q}_N}\}; \hat{z}_0, \overline{\hat{z}_0})]\}_{sym} \quad (9)$$

where $\delta_0 > 0$ is a parameter. The right side of (9) is symmetrized over all the sets of $(n - 1)$ zeros of the fermion Green function $R_L^f(\hat{z}, \hat{z}_0)$. By the above reasons, (9) vanishes, if Riemann surfaces become degenerated that provides the finiteness of the modular integrals in (1). It is follows from (3) and (4) that (9) is invariant under L_2 -transformations of \hat{z}_0 accompanied by suitable transformations of both $\lambda_j^{(1)}$ and $\lambda_j^{(2)}$. For a given $\hat{t}_0 = (\hat{z}_0 | \hat{\theta} = 0)$ one can calculate its image $\tilde{t} = (z_0 | \tilde{\theta}(z_0))$ under the mapping (2). In this case the transition functions have no poles because zeros $\hat{z}_j(\hat{z}_0; L)$ of $R_L^f(\hat{z}, \hat{z}_0)$ are always different from \hat{z}_0 . Simultaneously, so far as $\hat{z}_j(\hat{z}_0; L)$ is changed under fundamental group transformations, eqs.(3) are satisfied only if every this transformation is accompanied by an appropriate change of the $(\lambda_j^{(1)}, \lambda_j^{(2)})$ parameters that is calculated from (4). Because the above change of $(\lambda_j^{(1)}, \lambda_j^{(2)})$ does not depend on the superspin structure, (9) is invariant under the super-Schottky transformations of \tilde{t} .

3 Superstring amplitudes

To regularize the integrals over t_j , we need functions depending on two more supermanifold points $t_a = (t_{-1}, t_0)$ in addition to $\{t_j\}$. One receives in hands the above t_a points multiplying (1) by the unity arranged to be a square in the same integrals, every integral $I_{LL'}^{(n)} = 1$ being

$$I_{LL'}^{(n)} = \frac{1}{n} \int \frac{dt d\bar{t}}{2\pi i} I_{LL'}^{(n)}(t, \bar{t}), \quad I_{LL'}^{(n)}(t, \bar{t}) = D(t)[J_s(t; L) + \overline{J_s(t; L')}] [2\pi i \omega(L) - 2\pi i \overline{\omega(L')}]_{sr}^{-1} \times \overline{D(t)} [J_r(t; L) + \overline{J_r(t; L')}], \quad D(t) = \theta \partial_z + \partial_\theta. \quad (10)$$

Here $J_s(t; L)$ are the genus- n superholomorphic functions [8] having periods, $\omega_{sr}(L)$ is a period matrix on the supermanifold and $D(t)$ is the spinor derivative. The above period matrix depends on the superspin structure [5, 8]. Owing to $\overline{D(t)} J_r(t; L) = 0$, both $\overline{J_s(t; L')}$ and $J_r(t; L)$ can be omitted in (10), but we remain them to have the integrand without cuts on the supermanifold. Integrating (10) by parts one obtains that $I_{LL'}^{(n)} = 1$ as it was announced. With (10), we define a regularized superstring amplitude $A_{n,m}(\{\delta\})$ with $m > 3$ as

$$A_{n,m}(\{\delta\}) = \int \left(\prod_N dq_N d\bar{q}_N \right) \left(\prod_{r=1}^m dt_r d\bar{t}_r \right) \sum_{L, L'} \left(\prod_{a=-1}^0 dt_a d\bar{t}_a I_{LL'}^{(n)}(t_a, \bar{t}_a) \right) Z_{L, L'}^{(n)}(\{q_N, \bar{q}_N\}) \times E_{L, L'}(\{t_r, \bar{t}_r\}) B_{mod}^{(n)}(\{q_N, \bar{q}_N\}; \hat{z}_0, \bar{\hat{z}}_0; \delta_0) \prod_{(jl)} B_{jl}^{(n)}(\{t_a, \bar{t}_a\}; \{q_N, \bar{q}_N\}; \{\delta_{jl}\}; L, L') \quad (11)$$

where $t = (z_0 | \theta)$, the (jl) symbol labels pairs of the vertices, $\delta_{jl} > 0$ are parameters and $\{\delta\} = (\delta_0, \{\delta_{jl}\})$. Further, $\hat{z}_0 = \hat{z}_0(z_0)$ is calculated together with $\theta(z_0)$ from (2) taken at both $\hat{\theta} = 0$, $z = z_0$ and $\theta = \theta(z_0)$. At $\{\delta_{jl} > 0\}$ every $B_{jl}^{(n)}(\{t_a, \bar{t}_a\}; \{q_N, \bar{q}_N\}; \{\delta_{jl}\}; L, L')$ factor tends to zero at $|z_j - z_l| \rightarrow 0$ and at $|z_j - z_l| \rightarrow \infty$. The integration domain over moduli has been calculated in [13]. Also, one can rewrite (11) in the P_{split} variables. In this case moduli are integrated over the fundamental domain of the ordinary modular group. In details Grassmann integrations in (11) are planned to discuss elsewhere. The superstring amplitude $A_{n,m}$ is defined as $A_{n,m}(\{\delta \rightarrow 0\})$ calculated in line with the usual analytical continuation procedure [18] for the integrals over nodal domains giving rise to poles and threshold singularities of $A_{n,m}$. The regularization factors $B_{jl}^{(n)}(\{t_a, \bar{t}_a\}; \{q_N, \bar{q}_N\}; \{\delta_{jl}\}; L, L')$ are calculated in terms of the supermodular scalars $U_{jl}(\{t_a, \bar{t}_a\}; \{q_N, \bar{q}_N\}; L, L')$ defined by

$$U_{jl}(\{t_a, \bar{t}_a\}; \{q_N, \bar{q}_N\}; L, L') = \exp[2X_{j,l} + 2X_{-1,0} - X_{j,-1} - X_{j,0} - X_{l,-1} - X_{l,0}] \quad (12)$$

where $X_{r,s} \equiv X_{L, L'}(t_r, \bar{t}_r; t_s, \bar{t}_s)$ are the vacuum correlators of the scalar superfields. With (12), the desired factors in (11) can be constructed as

$$B_{jl}^{(n)}(\{t_a, \bar{t}_a\}; \{q_N, \bar{q}_N\}; \{\delta_{jl}\}; L, L') = \left[\frac{U_{jl}}{1 + U_{jl}^2} \right]^{\delta_{jl}^0} \exp[-\delta_{jl}^{(1)} U_{jl} - \delta_{jl}^{(2)} U_{jl}^{-1}] \quad (13)$$

where $U_{jl} \equiv U_{jl}(\{t_a, \bar{t}_a\}; \{q_N, \bar{q}_N\}; L, L')$ is given by (12). Grassmann integrations are well defined only for super-functions bounded together with all derivatives thereof [20] that is just provided by the exponential factor in (13). For $(p_j + p_l)^2 < 0$ we take $\delta_{jl}^0 = 0$. We define the

scalar product of 10-momenta by $p_j p_l = p_j^0 p_l^0 - p_j^n p_l^n$. So the factor in front of the exponential sign in (13) presents only, if $p_{jl}^2 = (p_j + p_l)^2 \geq 0$. Without the discussed factor nodal domain contributions to (11) that originate singularities of $A_{n,m}$ in p_{jl}^2 -variables would tend to infinity at $\{\delta \rightarrow 0\}$ because of terms $\exp[-p_{jl}^2 \ln \tilde{\delta}_{jl}]$ with $\tilde{\delta}_{jl} \in (\delta_{jl}^{(1)}, \delta_{jl}^{(2)})$ or $\tilde{\delta}_{jl} = \delta_0$. The factor in question presents, p_{jl}^2 is replaced by $p_{jl}^2 - \delta_{jl}^0$. So the discussed nodal domain integrations remain finite at $\{\tilde{\delta}_{jl} \rightarrow 0\}$, if $Re p_{jl}^2 < \delta_{jl}^0$. In this case the superstring amplitude $A_{n,m}$ are obtained from (11) by the following manifestly supermodular invariant procedure.

One calculates $A_{n,m}(\{\delta\})$ at $(\{\delta_{jl}^{(1)} = 0, \delta_{jl}^{(2)} = 0\}, \delta_0 = 0)$ in the domain where $Re p_{jl}^2 < \delta_{jl}^0$ for every p_{jl}^2 . If divergences are really cancelled in $A_{n,m}$, the result is finite due to (13). Next, for a particular p_{jl}^2 with $Re p_{jl}^2 > 0$, one performs an analytical continuation to $Re p_{jl}^2 < 0$. Thereafter δ_{jl}^0 is taken be zero. This procedure is performed step by step for every p_{rs}^2 with $Re p_{rs}^2 > 0$, the $A_{n,m}$ amplitude is obtained in a certain region of the $\{p_{jl}^2\}$ space.

4 Divergence cancellation and related topics

The procedure at the end of the previous Section verifies the supermodular invariance of $A_{n,m}$. In action all the δ_{jl} parameters can be nullify in (11) after a suitable rewriting of the integrals over the nodal domains following an appropriate super-Möbius transformation of $\{t_j\}$ (as it will be reported elsewhere). Really the factors (13) are important in (11) essentially to verify that the above rearrangement of (11) does not violate the supermodular group. Hence we expect that the gauge symmetry inherent to massless modes presents in $A_{n,m}$ though in (11) it is violated due to the discussed factors (13). A detailed study of the matter is in progress.

The 0-, 1-, 2- and 3- point functions do not depend on of 10-momenta of interacting particles. They are calculated from the factorization requirement on $A_{n,m}$ with $m > 3$ when a cluster $\{N_1\}$ of handles in a number of $n_1 < n$ is separated from the remainder $\{N_2\}$ of ones so that $d/l \rightarrow 0$. Here d is a size of the cluster and l is a typical distance between it and the remaining handles. In particular, 0-, 1-, 2- and 3-point functions of massless superstring modes can be defined to be zeros, if it is consistent with the above factorization requirement. By the grounds of the preceding paragraph one obtains from (11) that the discussed massless functions contribute to $A_{n,m}$ proportionally to either $A_{n_1,0}^{(n)}$ or $n_1 A_{n_1,1}^{(n)}$ to be

$$A_{n_1,1}^{(n)} = \int \left(\prod_{N_1} dq_{N_1} d\bar{q}_{N_1} \right) dt d\bar{t} \sum_{L_1, L'_1} Z_{L_1, L'_1}^{(n_1)}(\{q_{N_1}, \bar{q}_{N_1}\}) B_{mod}^{(n)}(\{q_N, \bar{q}_N\}; \hat{z}_0, \bar{\hat{z}}_0; \delta_0) I_{L_1 L'_1}^{(n_1)}(t, \bar{t}),$$

$$A_{n_1,0}^{(n)} = \int \left(\prod_{N_1} dq_{N_1} d\bar{q}_{N_1} \right) \sum_{L_1, L'_1} Z_{L_1, L'_1}^{(n_1)}(\{q_{N_1}, \bar{q}_{N_1}\}) B_{mod}^{(n)}(\{q_N, \bar{q}_N\}; \hat{z}_1, \bar{\hat{z}}_0; \delta_0) \quad (14)$$

where \hat{z}_1 a point distanced from the $\{N_1\}$ cluster in the order of l , superspin structures in the above cluster are labeled by (L_1, L'_1) and $I_{L_1 L'_1}^{(n_1)}(t, \bar{t})$ is defined in (10) at $n = n_1$. Further, $t = (z_0|\theta)$ and $\hat{z}_0 = \hat{z}(z_0)$ is calculated just as in (11). Through the regularization factor (9) the integrands in (14) depend on the variables associated with the remainder $\{N_2\}$ of the handles as well as on the $\{N_1\}$ cluster ones. The discussed functions of massless superstring modes are nullified as it is requested [3], if the right sides of eqs.(14) vanish at $(d/l \rightarrow 0, \delta_0 \rightarrow 0, d/l \gg \delta_0)$. A preliminary study gives evidence that this is really in the case as it is reported below.

Calculating (14) under conditions of interest we specify the $\{N_0\}$ set of the (3|2) super-Schottky parameters that are not moduli, say, as $N_0 = (u_1, v_1, u_2 | \mu_1 = 0, \nu_1 = 0)$ with a notation $(u_s | \mu_s)$ and $(v_s | \nu_s)$ for supercoordinates of two fixed points of the super-Schottky transformation associated with a given handle s . The right sides of (14) do not depend on (u_1, v_1) , as well as the right side of (11). Except only the $A_{1,0}^{(n)}$ case, the leading term of the exponent in (9) is invariant under a super-Möbius transformation of the super-Schottky fixed points assigned to the $\{N_1\}$ cluster. If the handle $s = 1$ does not belong to the above cluster, this transformation removes in (14) a dependence on two Grassmann variables. So both the integrals (14) are nullified owing to Grassmann integrations. In the $A_{1,0}^{(n)}$ case the leading term of the exponent in (9) does not depend on a spin structure assigned to $s = 1$. Hence $A_{1,0}^{(n)}$ vanishes due to a nullifying of the sum of the genus-1 partition functions. If $s = 1$ belongs to the $\{N_1\}$ cluster, one can simplify the right sides of (14) taking $|u_1 - v_1| \rightarrow 0$ with $|u_1 - v_1| \gg \delta_0 \rightarrow 0$. In so doing the leading term in $A_{1,0}^{(n)}$ and in $A_{1,1}^{(n)}$ is proportional to $A_{1,0}^{(n)}$ because the $s = 1$ handle is separated from the rest of the N_1 cluster and from \hat{z}_0 . Hence in this case the desired values disappear due to a vanishing of $A_{1,0}^{(n)}$. If the discussed values (14) vanish, a degeneration of the genus- n surfaces into a few ones of lower genus does not originate divergences in the $A_{n,M}$ amplitudes of interest. In addition, the divergences due to a degeneration of a particular handle are cancelled as it is usually [4, 13]. So the superstring amplitudes appear free from divergences. What values (14) vanish it was concluded from an examination only a leading term of the exponent in (9). An estimation of corrections it is yet necessary, especially, in regions where a number of zeros of the $R_L^f(\hat{z}, \hat{z}_0)$ fermion Green functions coincide. A detailed calculation of (14) is in progress.

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